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Microfunctions for boundary values problems

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1) Let X be a real manifold of class C^2, and let $D^+(X)$ denote the derived category of complexes of sheaves on X with cohomology bounded from below. Let $\pi: T^*X \to X$ denote the cotangent bundle to X.
In [K-S] we associate to $F \in \mathrm{Ob}(D^+(X))$ a closed subset $SS(F)$ of T^*X, the micro-support of F, and we also introduce the bifunctor μhom from $D^b(X)^\circ \times D^+(X)$ to $D^+(T^*X)$ which generalizes the functor μ_M of Sato's microlocalization along M. One has:

(1.1) $\mathrm{supp}(\mu hom(F,G)) \subset SS(F) \cap SS(G)$.

2) Let Ω be an open subset of X, $\bar{\Omega}$ its closure, $\partial\Omega = \bar{\Omega} \setminus \Omega$. We say that Ω is ℓct in X if:
(2.1) $\forall x \in \partial\Omega, (R\Gamma_{\bar{\Omega}}(\mathbb{Z}_X))_x = 0, (R\Gamma_\Omega(\mathbb{Z}_X))_x = \mathbb{Z}$.
Let $*$ denote the functor $R\,\underline{Hom}(\cdot, \mathbb{Z}_X)$. Then if Ω is ℓct, one has:

(2.2) $(\mathbb{Z}_\Omega)^* = \mathbb{Z}_{\bar{\Omega}}, (\mathbb{Z}_{\bar{\Omega}})^* = \mathbb{Z}_\Omega$.

Now let M be a closed submanifold of X . Assume:

(2.3) Ω is ℓct in X and $\bar{\Omega} \supset M$.

Then the morphism $\mathbb{Z}_{\bar{\Omega}} \to \mathbb{Z}_M$ defines by duality:

(2.4) $b: \mathbb{Z}_M \to \mathbb{Z}_\Omega \otimes \omega_{M/X}[d]$

where $\omega_{M/X}$ is the relative orientation sheaf and d is the codimension of M. We call b the "boundary value" morphism.

3) Assume now M is a real analytic manifold of dimension n and X is the complexification of M. Applying the functor $\mu hom(\cdot, \mathcal{O}_X)$ to (2.4) we get:

<u>Proposition 3.1</u> Let f be a holomorphic function on Ω. Then $b(f)$ is a well-defined hyperfunction on M, and

$$SS(b(f)) \subset T^*_M X \cap SS(\underline{\mathbb{Z}}_\Omega) .$$

4) Let ω be an open subset of M, j the inclusion map $\omega \to X$. We set:

(4.1) $C_{\omega|X} = \mu hom(\underline{\mathbb{Z}}_\omega , \mathcal{O}_X) \otimes \omega_{M/X} [n]$

There is a well-defined morphism:

(4.2) $\alpha: \; j_* j^{-1} \mathcal{B}_M \to \pi_* H^0(C_{\omega|X})$

If $\mathcal{M}$ is a coherent $\mathcal{D}_X$-module, there is a similarly defined morphism:

(4.3) $\alpha: \mathcal{E}xt^j_{\mathcal{D}_X}(\mathcal{M}, j_*j^{-1}\mathcal{B}_M) \to \pi_* H^j(R\underline{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{C}_{\omega|X})$

<u>Defunction 4.1</u> Let u be a section of $j_*j^{-1}\mathcal{B}_M$ (resp. of $\mathcal{E}xt^j_{\mathcal{D}_X}(\mathcal{M}, j_*j^{-1}\mathcal{B}_M))$. One sets: $SS_\omega(u) = \mathrm{supp}(\alpha(u))$

(resp.: $SS^{\mathcal{M},j}_\omega(u) = \mathrm{supp}(\alpha(u)))$.

There are similar definitions for microfunctions (using the morphism $\mathcal{C}_M \to \mathcal{C}_{\omega|X})$.

5) Let N be an analytic submanifold of M, Y the complexification of N in X .

Assume:

(5.1) Y is non characteristic for $\mathcal{M}$

(5.2) ω is ℓct in M and $\bar{\omega} \supset N$.

Using (2.4) one easily obtains a morphism

(5.3) $b: R\underline{H}om_{\mathcal{D}_X}(\mathcal{M}, j_*j^{-1}\mathcal{B}_M) \to R\underline{H}om_{\mathcal{D}_Y}(\mathcal{M}_Y, \mathcal{B}_N)$

This morphism extends the construction of [Sl] and [Ko], (cf. also [Ô]).

<u>Theorem 5.1</u> Let $u \in \mathrm{Ext}^j_{\mathcal{D}_X}(\mathcal{M}, \mathcal{B}_M(\omega))$. Then:

i) $SS_N^{\mathcal{M}_Y,j}(b(u)) \subset \rho\,\bar{\omega}^{-1} SS^{\mathcal{M},j}_\omega(u)$

ii) If $j = 0$, $\mathrm{codim}_M N = 1$, this inclusion is an equality.

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Here ρ and $\bar{\omega}$ denote as usual the map from $Y \times_X T^*X$ to T^*Y and T^*X, respectively. Remark that if $(C_{\omega|X})_{T^*_M X}$ is in degree 0 (e.g.: $\omega = M$ or else $\partial\omega$ is an analytic hypersurface). Then $SS_\omega^{M,0}(u) \cap T^*_M X = SS_\omega(u)$. In [S-Z] we study a notion of ω-regulariry for M which extends that of [S2] and [Ka] and which ensures that

$$SS_\omega(u) = \overline{SS(u|_\omega)} .$$

The results of this paper are detailed in [S3].

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